Shading

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Shading

- What can we learn about objects from their brightness?
  - Shape
  - Surface material properties
    - glossy, wet, rough, etc.

Sources, shadows and shading

- But how bright (or what colour) are objects?

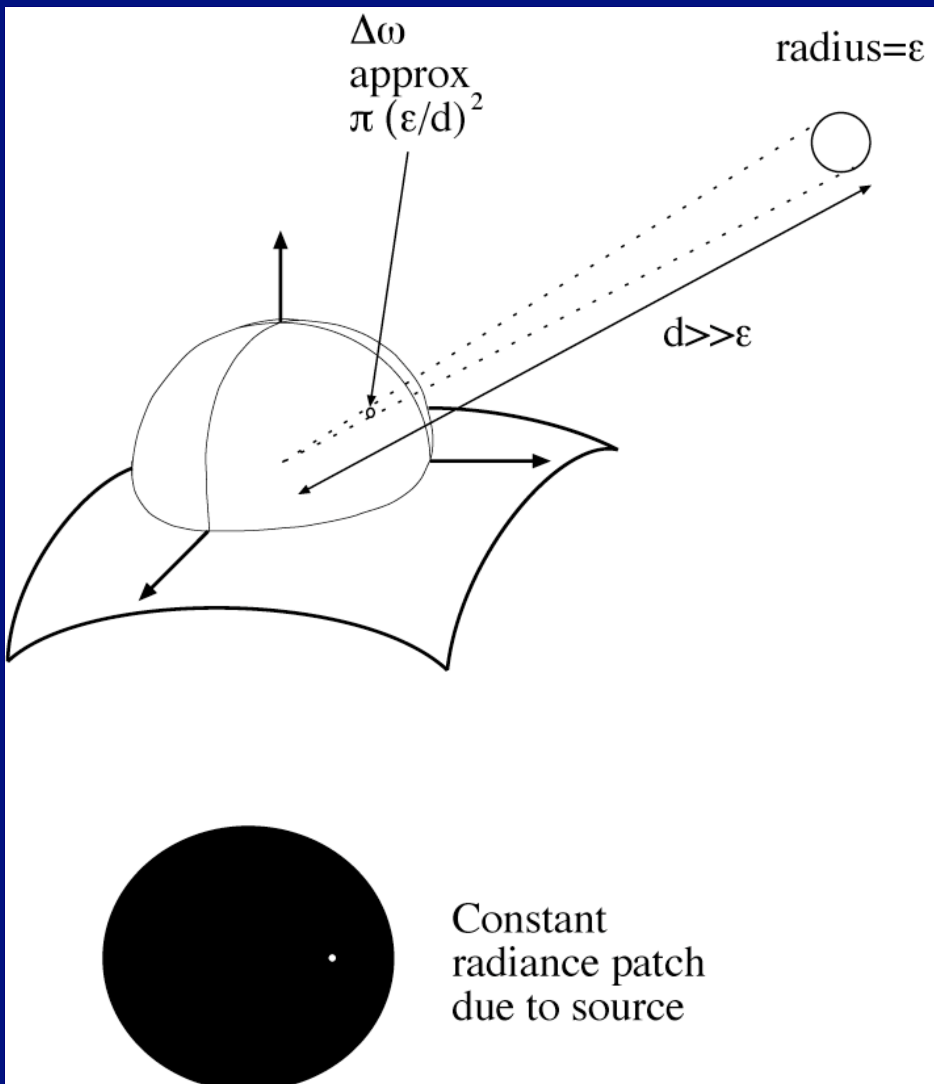
- For the moment, ignore specular components and think about Lambertian objects

$$B(\mathbf{x}) = E(\mathbf{x}) + \int_{\Omega} \left(\begin{array}{c} \text{Radiosity due to} \\ \text{incoming radiance} \end{array} \right) d\omega$$

Shading models

- Local shading model
  - Surface has radiosity due only to sources visible at each point
  - Advantages:
    - often easy to manipulate, expressions easy
    - supports quite simple theories of how shape information can be extracted from shading
- Global shading model
  - Surface has radiosity due to radiance reflected from other surfaces as well as from sources
  - Advantages
    - Very accurate, compelling
  - Disadvantage
    - Extremely difficult to infer anything from shading values

Radiosity due to point sources



• small, distant sphere radius ε and exitance E, which is far away subtends solid angle of about

$$\pi\left(\frac{\varepsilon}{d}\right)^2$$

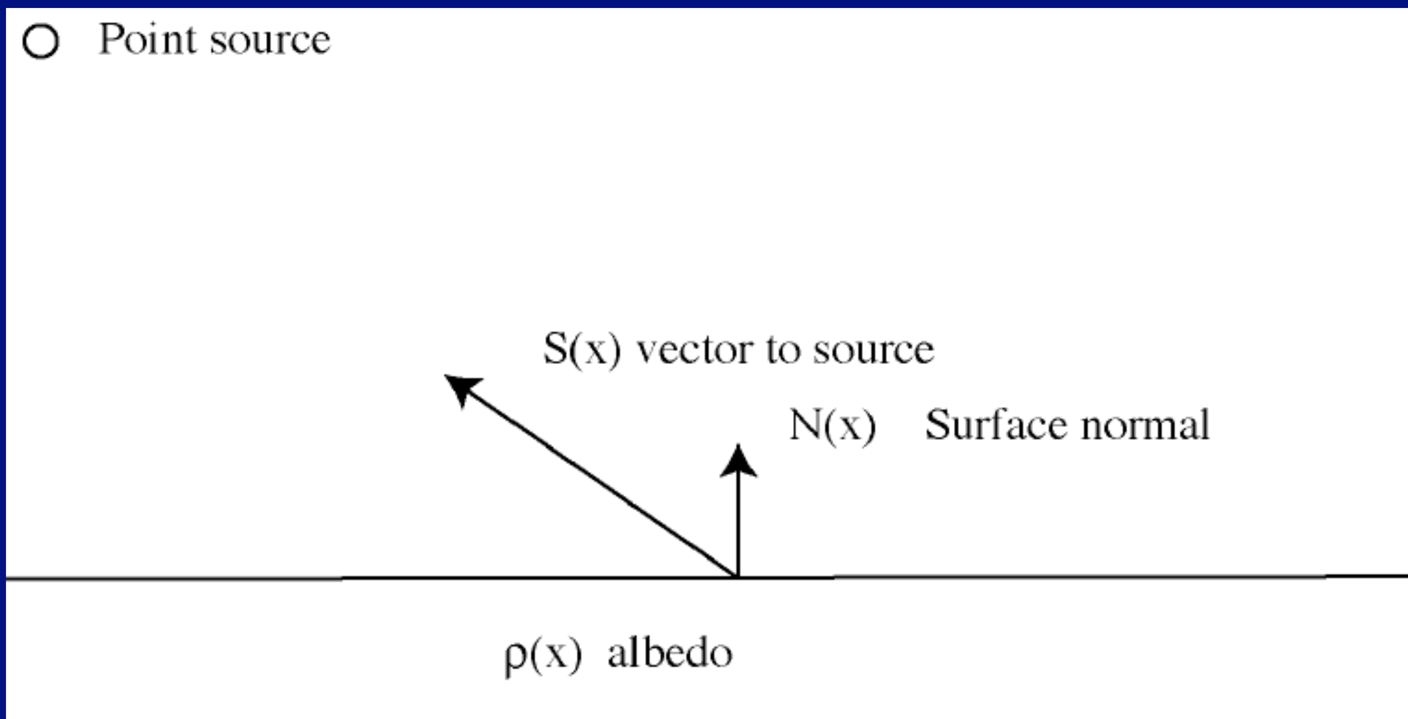
Radiosity due to a point source

- Radiosity is

$$
\begin{aligned}
B(x) &= \pi L_o(x) \\
&= \rho_d(x)\int_{\Omega} L_i(x,\omega)\cos\theta_i d\omega \\
&= \rho_d(x)\int_{D} L_i(x,\omega)\cos\theta_i d\omega \\
&\approx \rho_d(x)(\text{solid angle})(\text{Exitance term})\cos\theta_i \\
&= \frac{\rho_d(x)\cos\theta_i}{r(x)^2}(\text{Exitance term and some constants})
\end{aligned}
$$

Standard nearby point source model

$$I(\mathbf{x}) = \rho(\mathbf{x}) \frac{\mathbf{S}(\mathbf{x}) \cdot \mathbf{N}(\mathbf{x})}{r(\mathbf{x})^2}$$



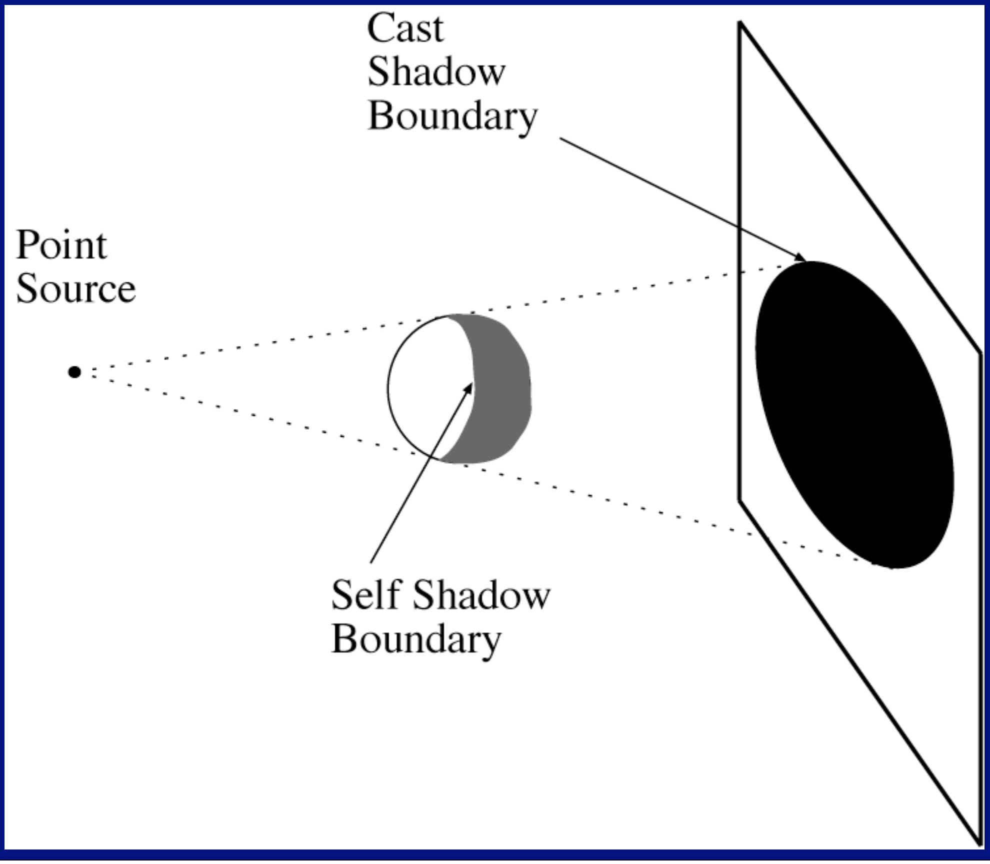
Standard distant point source model

- Issue: nearby point source gets bigger if one gets closer
  - the sun doesn't for any reasonable binding of closer
- Assume
  - all points in the model are close to each other with respect to the distance to the source. Then source vector doesn't vary much, and the distance doesn't vary much either, and we get

$$I(\mathbf{x}) = \rho(\mathbf{x})\mathbf{S} \cdot \mathbf{N}(\mathbf{x})$$

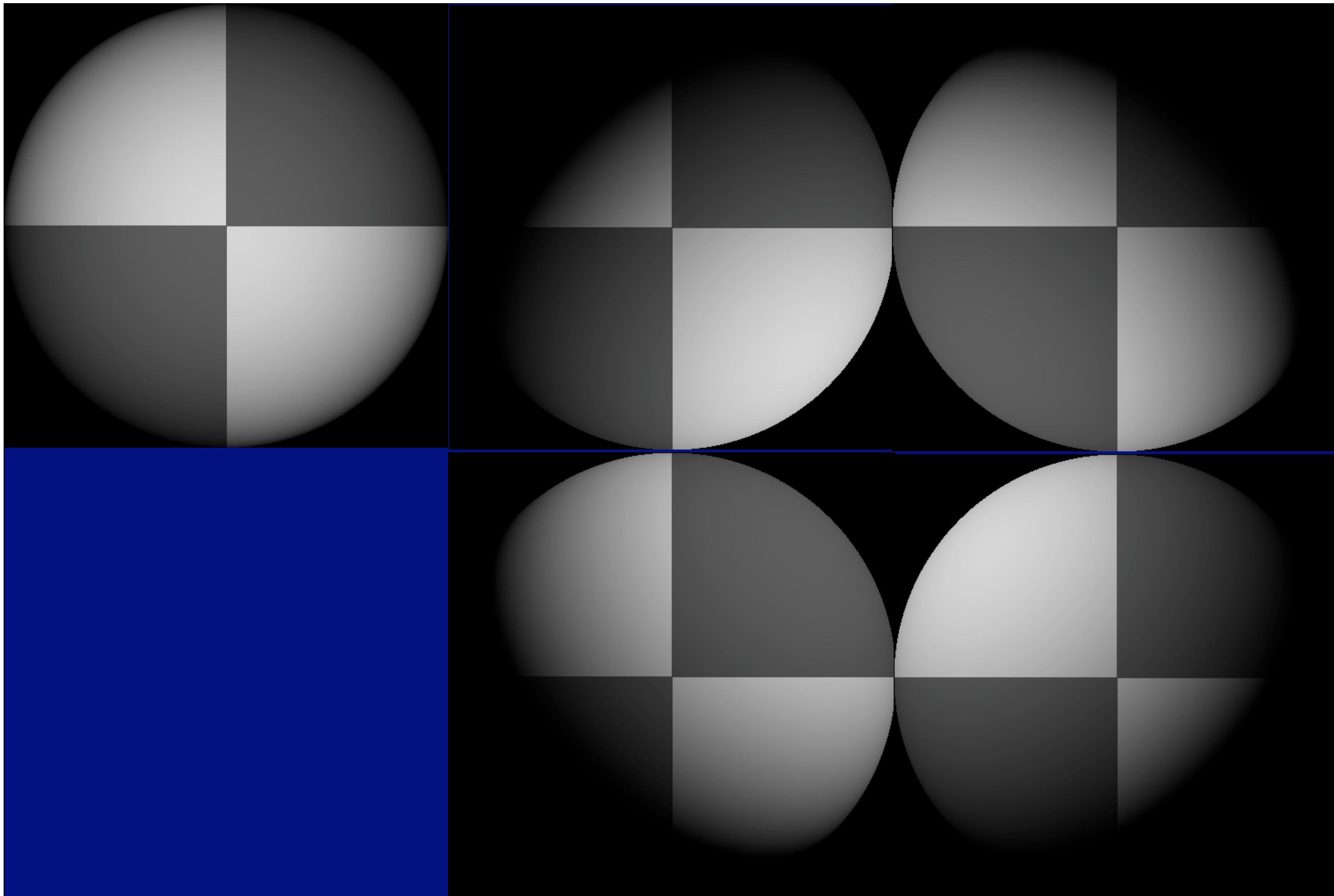
Shadows cast by a point source

- A point that can’t see the source is in shadow
- For point sources, the geometry is simple



Photometric stereo

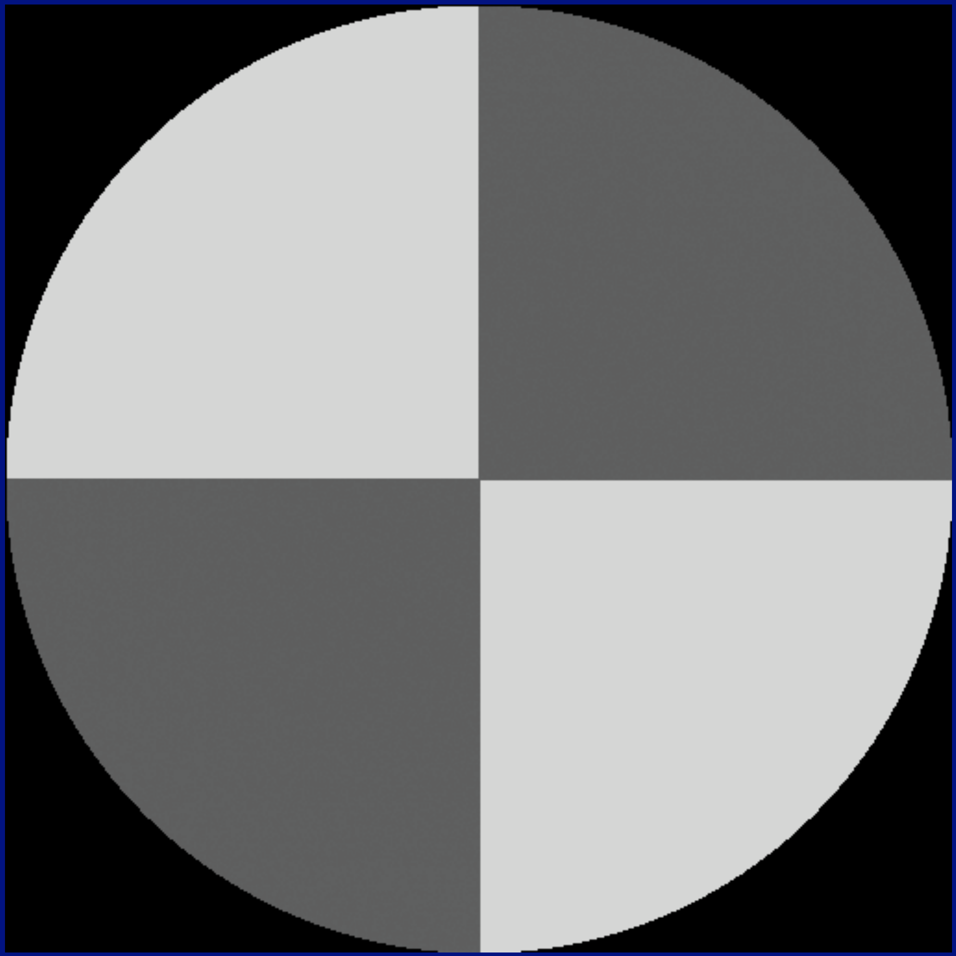
- Assume:
  - a local shading model
  - a set of point sources that are infinitely distant
  - a set of pictures of an object, obtained in exactly the same camera/object configuration but using different sources
  - A Lambertian object (or the specular component has been identified and removed)



Each image is: $$I_i(\mathbf{x}) = \mathbf{S}_i \cdot (\rho(\mathbf{x})\mathbf{N}(\mathbf{x}))$$

So if we have enough images with known sources, we can solve for

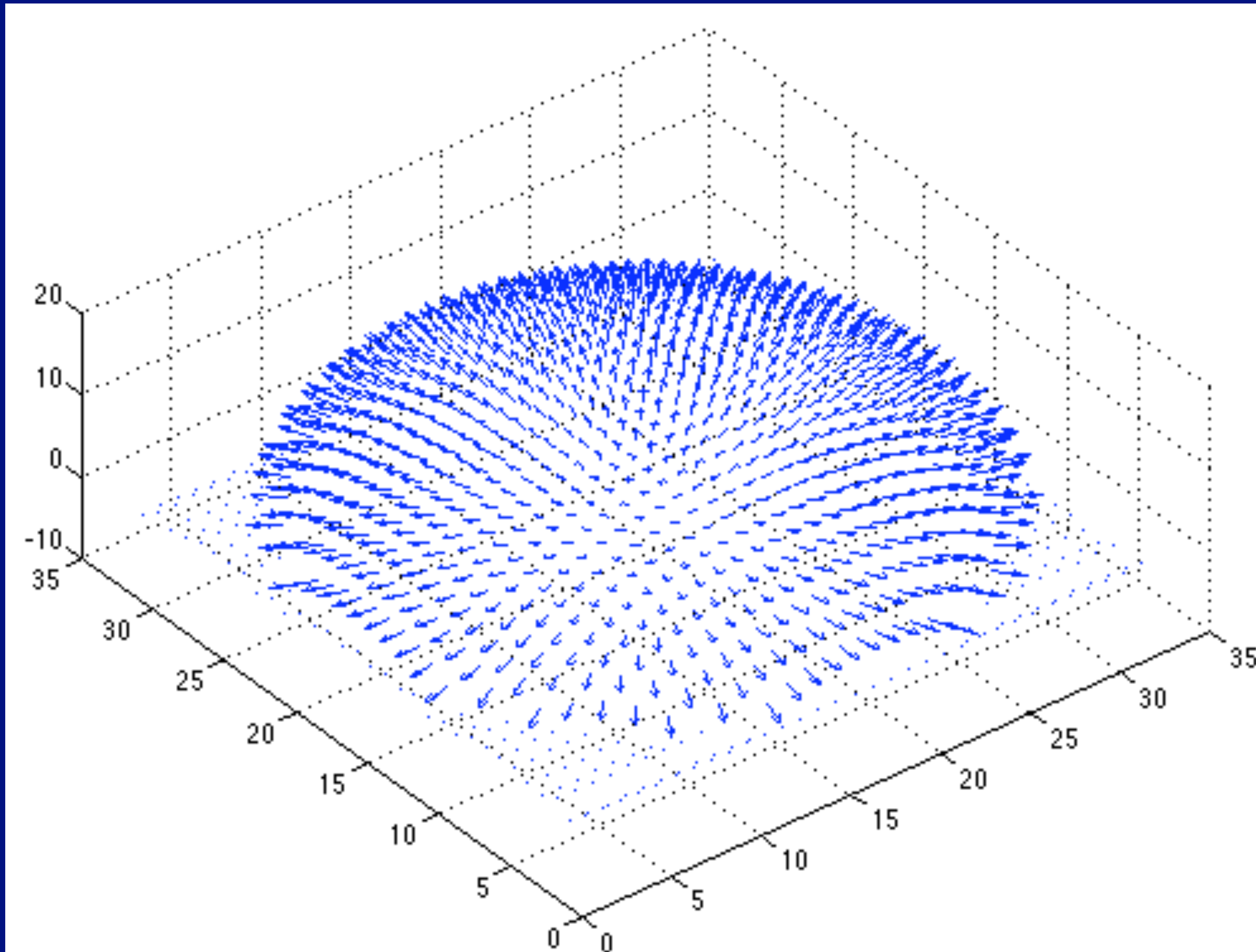
$$\mathbf{B}(\mathbf{x}) = (\rho(\mathbf{x})\mathbf{N}(\mathbf{x}))$$



$$\mathbf{B}(\mathbf{x}) = (\rho(\mathbf{x})\mathbf{N}(\mathbf{x}))$$

And the albedo (shown here) is given by:

$$\sqrt{\mathbf{B}(\mathbf{x}) \cdot \mathbf{B}(\mathbf{x})}$$



20
10
0
-10
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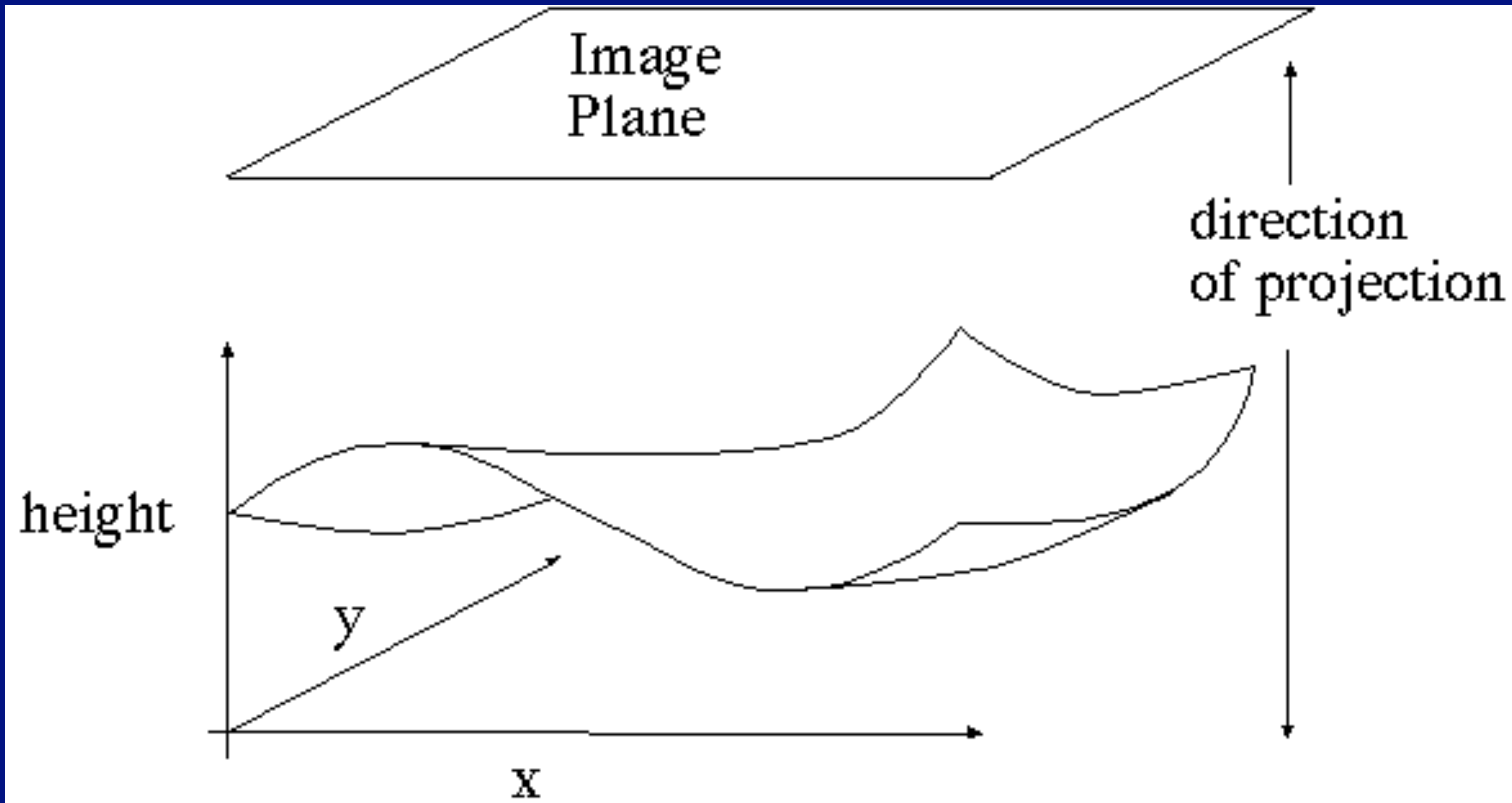
20
25
30
35

Projection model for surface recovery



Recovering the surface

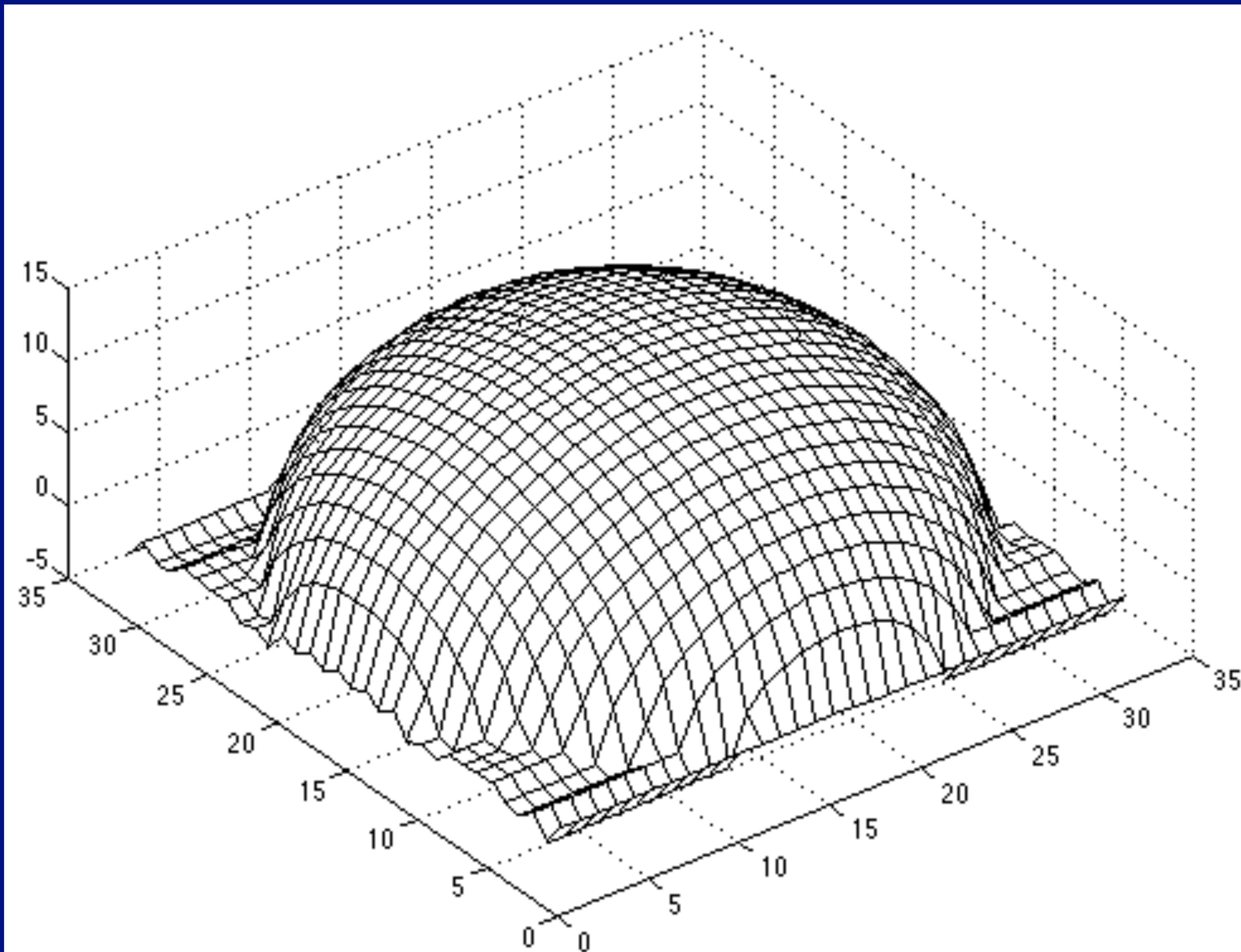
Surface is: $(x, y, f(x, y))$

Normal of this surface is:

$$\frac{(-\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 1)}{\sqrt{1 + \frac{\partial f}{\partial x}^2 + \frac{\partial f}{\partial x}^2}}$$

And this means that

$$\frac{\partial f}{\partial x} = \frac{-B_1(\mathbf{x})}{B_3(\mathbf{x})} \qquad \frac{\partial f}{\partial y} = \frac{-B_2(\mathbf{x})}{B_3(\mathbf{x})}$$



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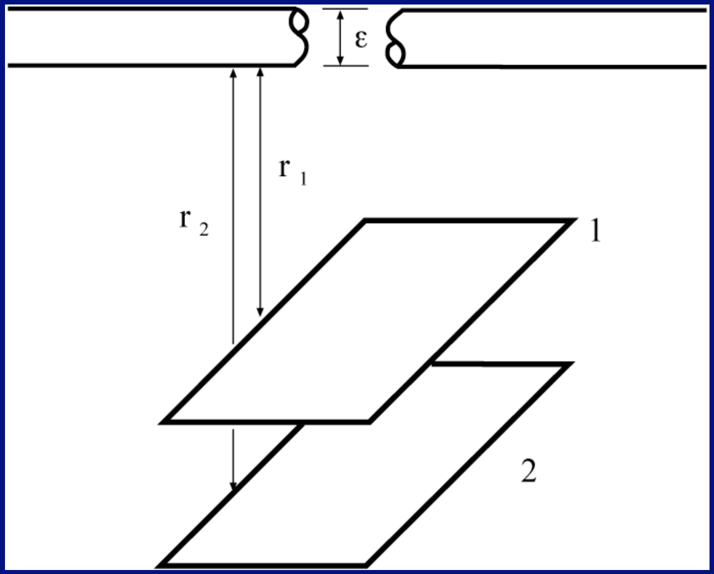
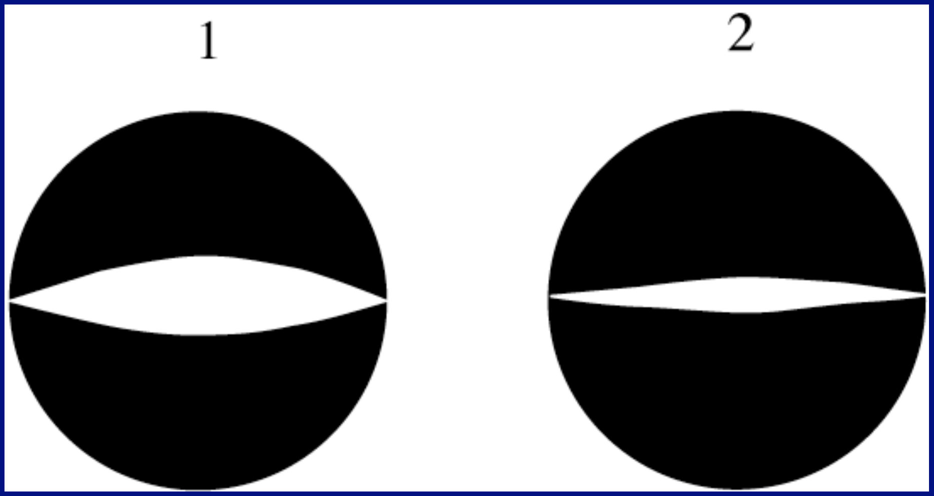
Line sources



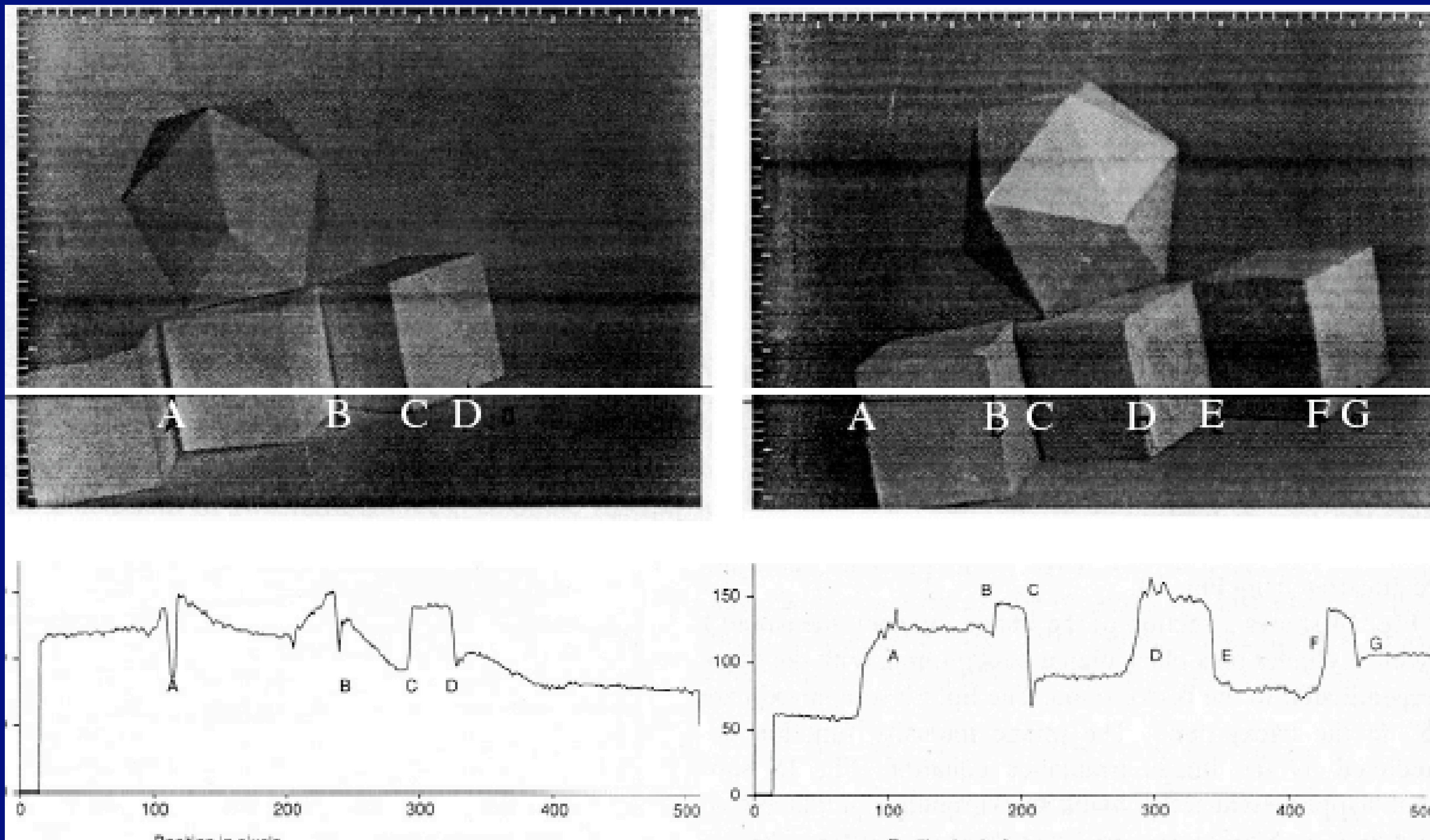


radiosity due to line source varies with inverse distance,
if the source is long enough

Curious Experimental Fact

- Prepare two rooms, one with white walls and white objects, one with black walls and black objects
- Illuminate the black room with bright light, the white room with dim light
- People can tell which is which (due to Gilchrist)

- Why? (a local shading model predicts they can't).


A B C D
A B C D E F G
A
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Position in pixels
150
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50
0
A
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E
F
G
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100
200
300
400
500
Position in pixels

Interreflections

- Issue:
  - local shading model is a poor description of physical processes that give rise to images
    - because surfaces reflect light onto one another
  - This is a major nuisance; the distribution of light (in principle) depends on the configuration of every radiator; big distant ones are as important as small nearby ones (solid angle)
  - The effects are easy to model
  - It appears to be hard to extract information from these models

Sources, shadows and shading

$$B(x) = E(x) + \int_{\Omega} \left\{ \begin{array}{l} \text{radiosity due to} \\ \text{incoming radiance} \end{array} \right\} d\omega$$

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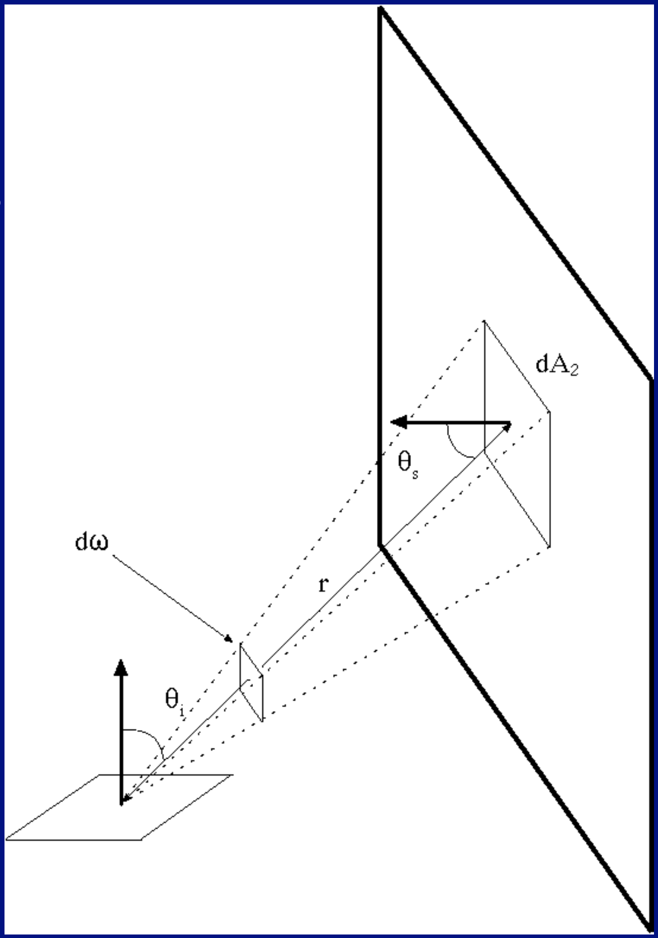
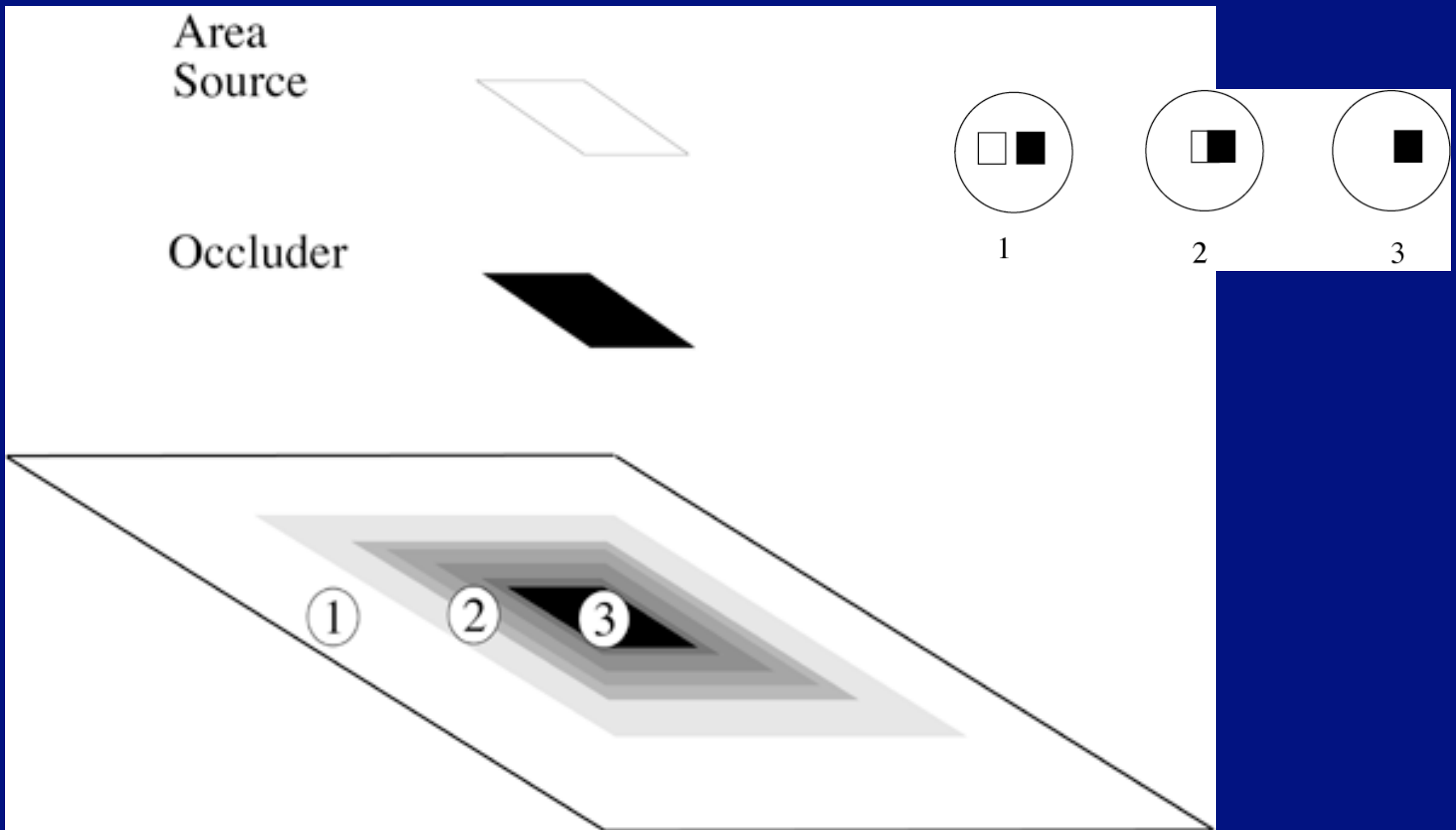
Area sources



- Examples: diffuser boxes, white walls.
- The radiosity at a point due to an area source is obtained by adding up the contribution over the section of view hemisphere subtended by the source
  - change variables and add up over the source

Area Source Shadows



Radiosity due to an area source

- rho is albedo
- E is exitance
- r(x, u) is distance between points
- u is a coordinate on the source

$$B(x) = \rho_d(x)\int_{\Omega} L_i(x, u \to x)\cos\theta_i d\omega$$

$$= \rho_d(x)\int_{\Omega} L_e(x, u \to x)\cos\theta_i d\omega$$

$$= \rho_d(x)\int_{\Omega}\left(\frac{E(u)}{\pi}\right)\cos\theta_i d\omega$$

$$= \rho_d(x)\int_{source}\left(\frac{E(u)}{\pi}\right)\cos\theta_i\left(\cos\theta_s\frac{dA_u}{r(x,u)^2}\right)$$

$$= \rho_d(x)\int_{source} E(u)\frac{\cos\theta_i\cos\theta_s}{\pi r(x,u)^2}dA_u$$

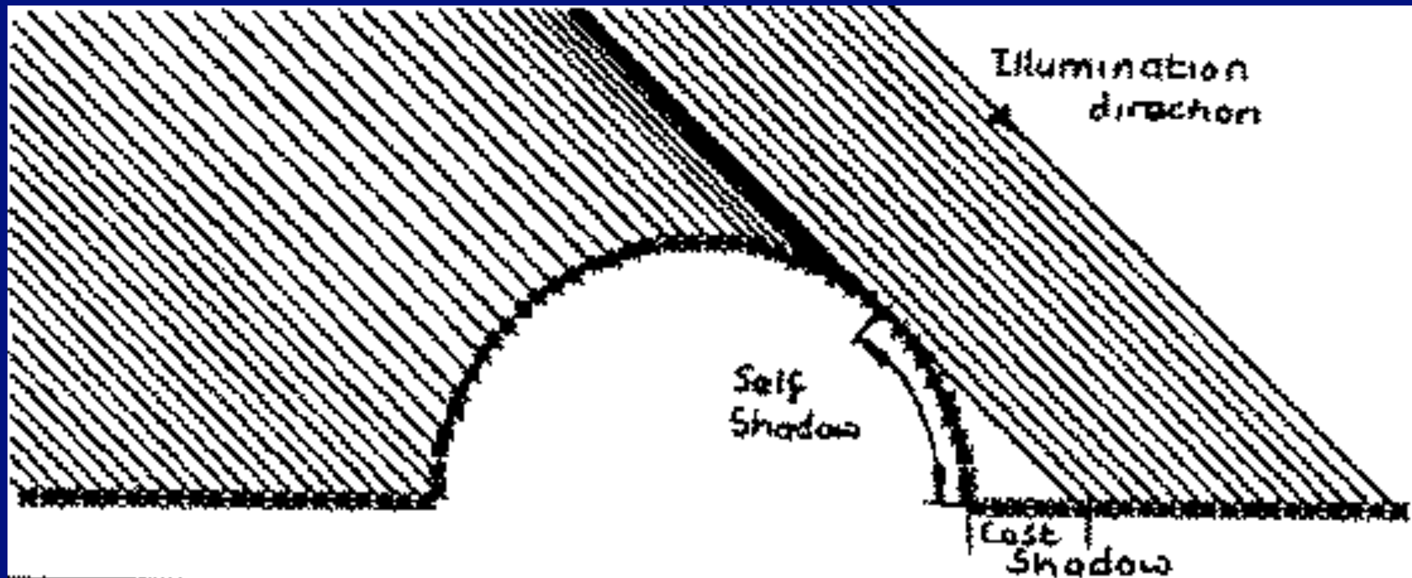
Interreflections

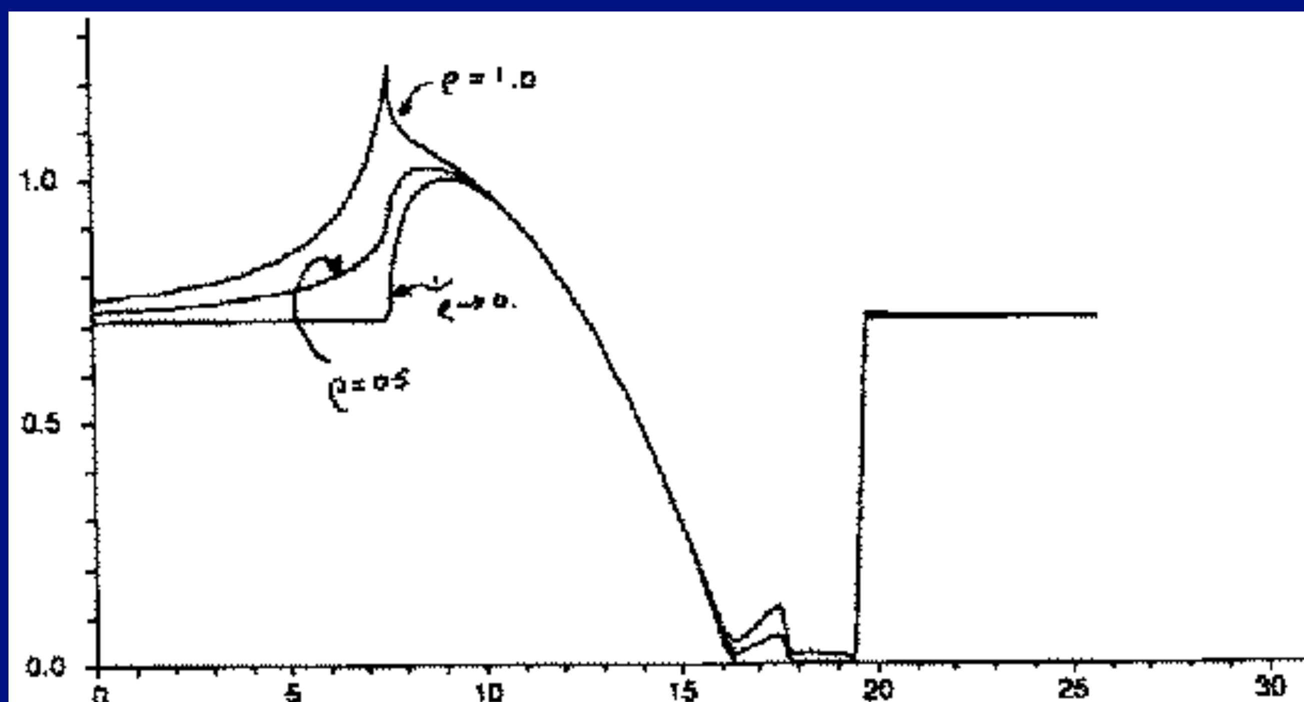
- Radiosity at surface=Exitance plus radiosity due to incoming radiosity from all other surfaces
- This gives an integral equation (below)
- Vis(x, u) is 1 if they can see each other, 0 if they can't
- Well understood by the graphics community, with many tricks known

$$B(x) = E(x) + \rho_d(x) \int_{\text{all other surfaces}} B(u) \frac{\cos\theta_i \cos\theta_s}{\pi r(x,u)^2} Vis(x,u) dA_u$$

What do we do about this?

- Attempt to build approximations
  - Ambient illumination
- Study qualitative effects
  - reflexes
  - decreased dynamic range
  - smoothing
- Try to use other information to control errors


Illumination direction
Self Shadow
Cast Shadow


1.0
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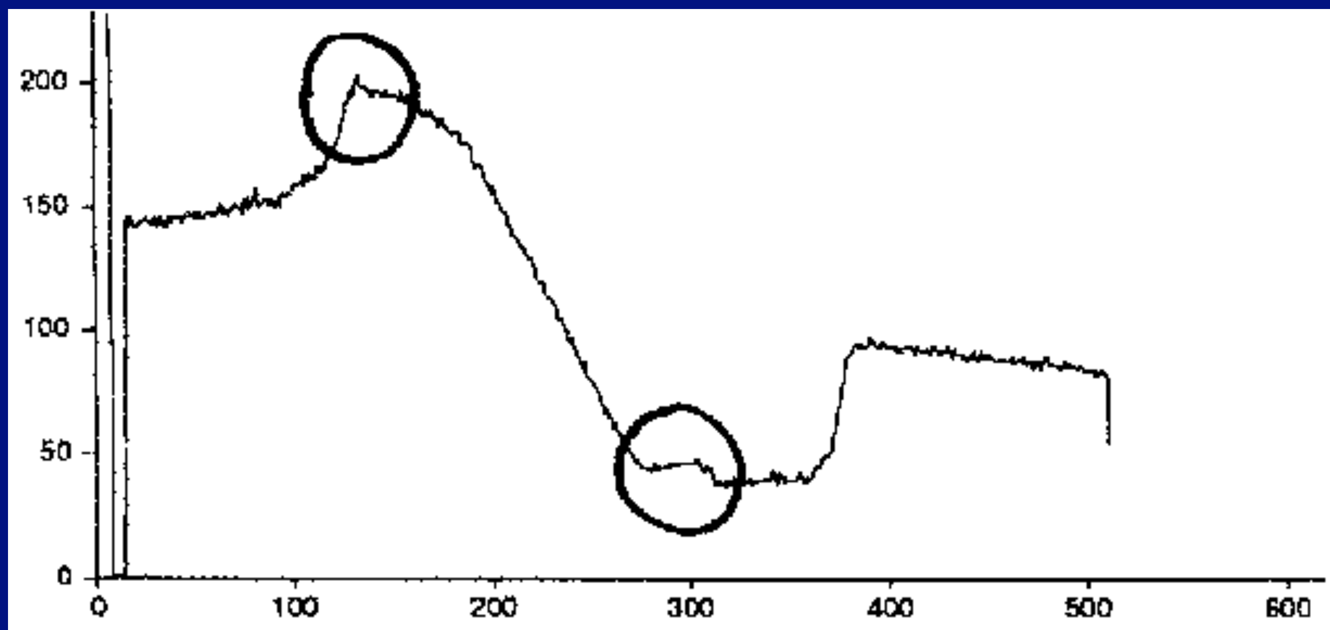
200
150
100
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0
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100
200
300
400
500
600

Specularity detection

- Specularities are:
  - small
  - very bright
  - a different color from surface
- This yields several different detectors